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MECHANICS OF MANIFOLD FLOW

by John S. McNown

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MECHANICS OF MANIFOLD FLOW

by

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INTRODUCTION

Changes in piezometric and total head at branch points are significant design criteria for various kinds of manifold systems. Dividing flows take place in sprinkler irrigation systems and in gas-burner manifolds, and both combining and dividing flows occur in lock manifolds and in complex water supply systems. The complexity of the flow patterns at branch points precludes rigorous analysis, but a general understanding can be obtained from a combination of experimental results with those of simplified analyses. Although the geometry of the conduit and the junction is infinitely variable, the significant characteristics of manifold flow can be determined from a study of an idealized form, as indicated in the following discussion of combining and dividing flow in circular conduits.

From the designer's point of view, any problem breaks down into the prediction of (a) the losses in regions of parallel flow, and (b) the changes in pressure and the losses at the junction. In some instances the branch points are so close together that mutual interactions of successive junctions affect these values; a useful simplification is nonetheless effected by considering a single branch point in a conduit in which the flow is otherwise uniform throughout a considerable distance upstream and downstream from the junction.

As there is a marked difference between dividing and combining flows, the analyses and results for the two cases will be presented separately. In every instance the diameter of the main pipe and the direction of flow therein are taken to be the

same both upstream and downstream from the junction, and the diameter of the lateral pipe is equal to or less than that of the main pipe. The axes of the lateral and of the conduit are considered to intersect at right angles.

As a project of the Iowa Institute of Hydraulic Research, manifold flow was studied in a series of Master's theses performed by Barton [1]*, Niaz [2], Escobar [3], and Yanes [4]; the experimental work presented herein was conducted by these men under the writer's supervision. Funds for equipment for this project in its early stages were provided by the American Society of Civil Engineers through the Engineering Foundation.

During the past 40 years studies of this topic have been made by investigators working in several different fields. In an excellent series of experiments at Munich [5, 6, 7], Thoma's students conducted a systematic investigation of this phenomenon. Publications have been presented on the general problem of efflux from a perforated pipe, one of the more recent being that by Howland [8]. This phase of manifold flow was also summarized in connection with irrigation systems by Christianson in 1942 [9]. A recent and comprehensive study of gas-burner manifolds was presented by Keller [10]. Various aspects of the flow in lock manifolds have been described by Soucek and Zelnick [11] and Rich [12], among others. Although in most of these publications the conclusions reached were founded on experiment, useful analyses based on simplified versions of the momentum and energy principles have been presented by several of the aforementioned writers and by Favre [13]. The writer with Hsu [14] presented an entirely different type of analysis using the free-streamline theory in determining the principal characteristics of the lateral efflux. The various papers cited contain references to a number of others, among which one finds diversified information which is, for the most part, either specialized or fragmentary.

THEORY

The energy equation can be written for both the main conduit flow and for that in the lateral if a term is included for the

* Numbers in brackets refer to References at end of paper.

energy loss. From a similarly simplified point of view, one can write the momentum equation for the flow at the junction, provided that a term is included for (a) the indeterminate momentum of the flow in the lateral at the junction, or (b) the corresponding unbalanced force component on the wall of the lateral. In either equation, lack of knowledge of this one significant unknown makes direct application of the results impossible without recourse to experiment. The results obtained are nonetheless useful in the assessing of the observed characteristics of branching flow.

Dividing Flow

Simplified forms of the energy and momentum equations can be written directly for dividing flow. The assumption is made that the mean velocity at each section (Fig. 1a) is representative of the

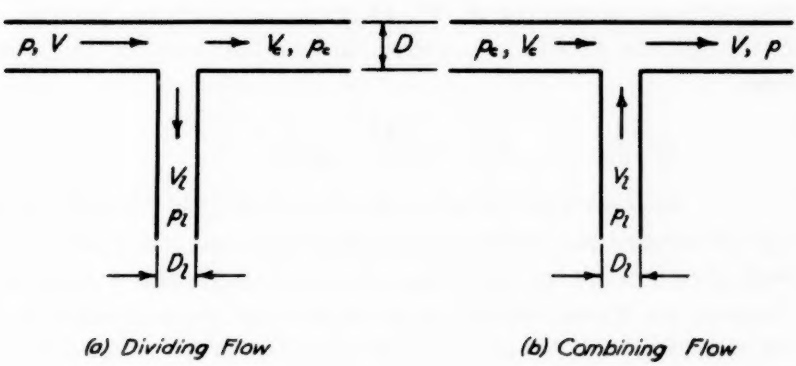

Fig. 1. Definition sketch for manifold flow.

flow at that section. Also, terms for the head losses are taken to represent the difference between the total losses and the losses observed in normal conduit flow. For the flow that continues in the conduit, the change in piezometric head Δh is expressible in the following forms:

$$\frac{\Delta h}{V^2/2g} = \frac{p_c - p}{\rho V^2/2} = 1 - \left(\frac{V_c}{V}\right)^2 - \frac{h_f}{V^2/2g}$$

$$= \frac{Q_l}{Q}\left(2 - \frac{Q_l}{Q}\right) - \frac{h_f}{V^2/2g} \qquad (1)$$

For the flow into the lateral, a similar expression can be obtained, but for reasons which will be presented subsequently, the various terms are related to the velocity head in the lateral:

$$\frac{\Delta h'}{V_l^2/2g} = \frac{p_l - p}{\rho V_l^2/2} = \left(\frac{V}{V_l}\right)^2 - 1 - \frac{h_f'}{V_l^2/2g} \qquad (2)$$

In the momentum equation it is necessary to include the resultant of the unbalanced pressures inside the lateral. As the direction of flow of the fluid going into the lateral is inclined to the axis of the lateral, either the component of the resultant momentum in the direction of conduit flow or the force required to reduce it to zero must be included in a bulk momentum equation. If this force is represented by F , and if it is considered positive in the upstream direction, the momentum equation takes the following form:

$$\frac{\Delta h}{V^2/2g} = 2\left[1 - \left(\frac{V_c}{V}\right)^2 - \frac{F}{\rho Q V}\right] \qquad (3)$$

An exact solution was obtained in 1951 [14] for the special case of irrotational two-dimensional branching flow, the classical Helmholtz-Kirchhoff theory of free streamlines providing a means of obtaining the characteristics of the efflux for various geometries and flow patterns. The numerous uncertainties in the applying of the results of such calculations to flows of real fluids in circular conduits and with wholly submerged efflux are evident. Only if such a theory provides results which coincide with those observed in the laboratory can it be used as a basis for prediction.

This application of irrotational-flow theory in the solution of this type of problem [14] is somewhat novel, particularly as it was used for the calculation of a head-loss term, $h_f'/(V_l^2/2g)$. By means of the method of successive conformal transformations, a relationship was obtained for the theoretical coefficient of con-

traction for two-dimensional lateral efflux. With reference to Fig. 2, the relationship

$$C_c = f\left(\frac{V_c}{V}, \frac{a}{b}\right) \tag{4}$$

was defined in general but implicit form, and representative curves were presented.

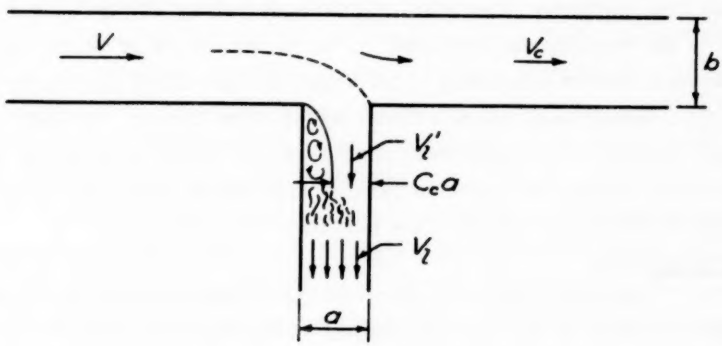

Fig. 2. Flow pattern for two-dimensional branch point.

The results represented by Eq. (4) were assumed to be applicable to the divided flow of a real fluid in circular conduits with the provisions that (a) the ratio of the area of the lateral to the area of the main conduit was the same in each case, and (b) the energy loss took place as in an abrupt expansion downstream from some point at which the contraction of the jet could be assumed essentially complete. Thus, C_c was obtained for a value of a/b equal to $(D_l/D)^2$, and h_f' was then computed from the known formula for head loss at a boundary enlargement,

$$h_f' = \frac{(V_l' - V_l)^2}{2g}$$

in which V_l' is the velocity at the contracted section $Q_l/C_c A_l$.

It follows that the head loss for the flow in the lateral is

$$\frac{h_f'}{V_l^2/2g} = \left(\frac{1}{C_c} - 1\right)^2 \tag{5}$$

One flaw in the analysis leading to Eq. (4) is that the stagnation point for the dividing streamline was forced to fall at the downstream edge of the opening regardless of the geometry and discharge ratio. As a consequence, a fictitious partial barrier projecting into the flow was artificially introduced into the computed flow patterns. For most significant values of Q_l/Q and a/b , the barrier was very small and, on the basis of other calculations, not particularly significant in its effect on the value of C_c . Additional calculations, as yet unpublished, indicate the manner in which the downstream channel width should be altered to cause the stagnation point to fall at the proper point without artificial modification of the flow pattern.

Combining Flow

Markedly different from dividing flow in physical characteristics, combining flow is nonetheless describable in general equation form by means of Eqs. (1) to (3) with only minor modifications. The total flow Q occurs downstream from the junction in this case, the subscript c is used for the conduit flow upstream, and the direction of lateral flow is reversed (Fig. 1b). As p_c is again greater than p , it is logical, if not conventional, to leave the equations in the forms given; only the sign of the last term in each equation must then be changed. If Δh is now $(p_c-p)/\gamma$

$$\frac{\Delta h}{V^2/2g} = 1 - \left(\frac{V_c}{V}\right)^2 + \frac{h_f}{V^2/2g} \tag{6}$$

Similarly, if F is again taken to be positive in the upstream direction, the sign on the term $F/(\rho QV)$ in Eq. (3) should be positive for combining flow:

$$\frac{\Delta h}{V^2/2g} = 2\left[1 - \left(\frac{V_c}{V}\right)^2 + \frac{F}{\rho QV}\right] \tag{7}$$

The irrotational-flow theory comparable to that presented for divided flow has not been found to give useful results. The

difference between the idealized flow pattern for plane motions and the real conduit flow is apparently considerably greater in this case. In the analysis, it is assumed that the two jets coalesce with the entire downstream flow separating from the boundary just as did the lateral flow in the preceding case. For this model a coefficient of contraction and a head loss can again be computed. In an unpublished calculation, the losses thus computed were found to differ greatly from those measured by Niaz for flow in circular conduits. It was concluded that the marked differences arose because the two jets did not coalesce as assumed. That is, particularly for laterals smaller in diameter than the main conduit, the lateral jet presumably pierced the main jet leaving part of the conduit flow comparatively undisturbed, a state of flow for which the assumed mathematical model is quite inadequate.

The various equations have been presented in the foregoing paragraphs both to serve as a framework for the presentation and discussion of the experimental data, and to make possible the comparing and assessing of various simplified and therefore approximate analyses. Enger and Levy [15] proposed the neglect of the head-loss term in Eq. (1) and supported their idea by data from experiments for which D_l/D was quite small. Soucek and Zelnick [11] and several discussers of that paper, including the writer, suggested and compared various assumptions for the indeterminate quantities of head loss and unbalanced force. Favre, for combining flow, assumed the unbalanced force to be zero, and presented a detailed treatment of this kind of flow [13]. His assumption was found to give results which corresponded well with those of the Munich experiments. Actually, as is shown in the following sections, no simplified analysis is valid throughout a sufficiently large range of the significant variables to have general significance beyond that of a reference for the comparison and evaluation of laboratory measurement.

LABORATORY INVESTIGATION

In the absence of a general analysis of manifold flow, laboratory studies must be conducted to augment the available knowledge

of this subject. Because the geometry in each application differs from almost every other, no exhaustive study of the various possible arrangements is feasible. General information can only be obtained by determining the characteristics of flow for a few manifolds of simple geometric form. The studies conducted at the Iowa Institute of Hydraulic Research, described in the following paragraphs, are comparable in this respect to those conducted at Munich. A greater number of basic manifold shapes were investigated in the experiments in Germany, but considerably more detailed results obtained in the Iowa experiments make possible a more nearly complete interpretation of the phenomenon.

Throughout this study, a 2-inch brass pipe (actual inside diameter 2.06 inches) was used as the main conduit. A similar pipe and lengths of brass tubing 1 inch and 1/2 inch in diameter were used as laterals. All sections of pipe and tubing employed were between 75 and 100 diameters long. The inside surfaces were smooth except for slight traces of chemical deposit. The three junctions were machined Tees, the intersection being sharp-edged.

For two special series of experiments, additional junctions were made. A second junction with a 1-inch lateral was constructed so that the effect of one lateral on another a short distance downstream could be studied. The second lateral and a short spacer were so constructed that the two could be placed 4, 8, or 14 conduit diameters apart. For the other set of additional experiments, a junction was made in the form of a cross with two 1-inch laterals being symmetrically placed on opposite sides of the conduit.

Water was supplied to the various pipe arrangements from a constant-level tank through a stilling tank (two tanks in the case of combining flow). The flows were varied by changing valve settings either upstream from the stilling tanks or downstream from the conduit and lateral. Streamlined entrance sections were provided and the two parts of the flow were measured either by direct weight or by a combination of direct weight and observations of a calibrated discharge meter at the inlet contraction. The equipment was arranged

so that the total discharge could be measured independently and compared with the sum of the two component discharges as a check. Piezometer openings were provided at intervals of about 5 tube diameters for determination of the three pressure gradients. In observing the various piezometric heads, a pressure manifold and either a mercury-water or an air-water differential gage were employed, readings being made to the nearest 0.001 foot.

Before measurements were made of the manifold flows the head loss for uniform flow was determined for each of the various

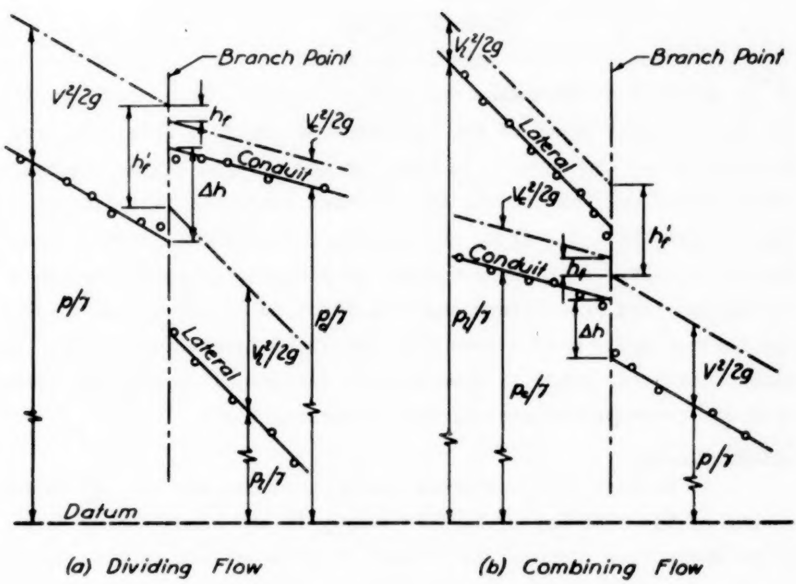

Fig. 3. Results of typical experiment.

pipe sections. Plots were prepared of the piezometric-head gradient against the discharge, so that the apparent gradient, as observed during individual runs, could be checked for each section of the pipe in every run. In this way large errors were eliminated, and

the spread of the data was considerably reduced.

After each run the observed piezometric heads were plotted for the three sections of pipe, and straight lines of correct slope were then drawn through the sets of points in the region of undisturbed flow of each pipe. Runs for which the gradients were incorrect or poorly defined were discarded. The lines were then extended to the branch point as indicated by the solid lines in Fig. 3. From these plots and from the observed discharge values the velocity heads, the total head (dash-dot lines), and the two head losses were computed as also indicated schematically if Fig. 3. Complete details of equipment and procedure are available in thesis form [1-4].

OBSERVED RESULTS

From the tests made as described in the foregoing section, it is possible to determine the change in piezometric head for each of the component parts of the two types of manifold flow: (1) for dividing flow, the changes in head for the portion of the discharge continuing along the conduit and for that going into the lateral; and (2) for combining flow, the conduit flow and that from the lateral to the conduit. Once these changes in piezometric head ascribable to the junction were determined as a function of Q_l/Q and D_l/D, the corresponding head losses and unbalanced forces were known. As each of the four kinds of flow differs fundamentally from the others, they are presented separately and in various forms.

Dividing Flow

Because many different design problems involve the determining of the recovery of kinetic energy in the conduit flow at a point where flow divides, this phase of the manifold flow is probably the most important. The results obtained by Barton at Iowa [1] are shown in Fig. 4 for D_l/D ratios of 1, 1/2, and 1/4. Together with them are mean curves presented by Vogel from the Munich experiments [5] for D_l/D ratios of 1 and 0.58. Vogel also made measurements for $D_l/D = 0.35$, but he indicated that the scatter of points was so great that no mean trend was defined. Except for a discrepancy in the region of the uppermost portion of the curves for $D_l/D = 1$,

the two series of experiments indicate almost identical results.

Also presented for comparison in Fig. 4 are two curves calculated from Eq. (1), one for a zero head loss and the other for a head loss equal to that in a sudden expansion. The latter is included because the decrease in velocity for the conduit flow at the

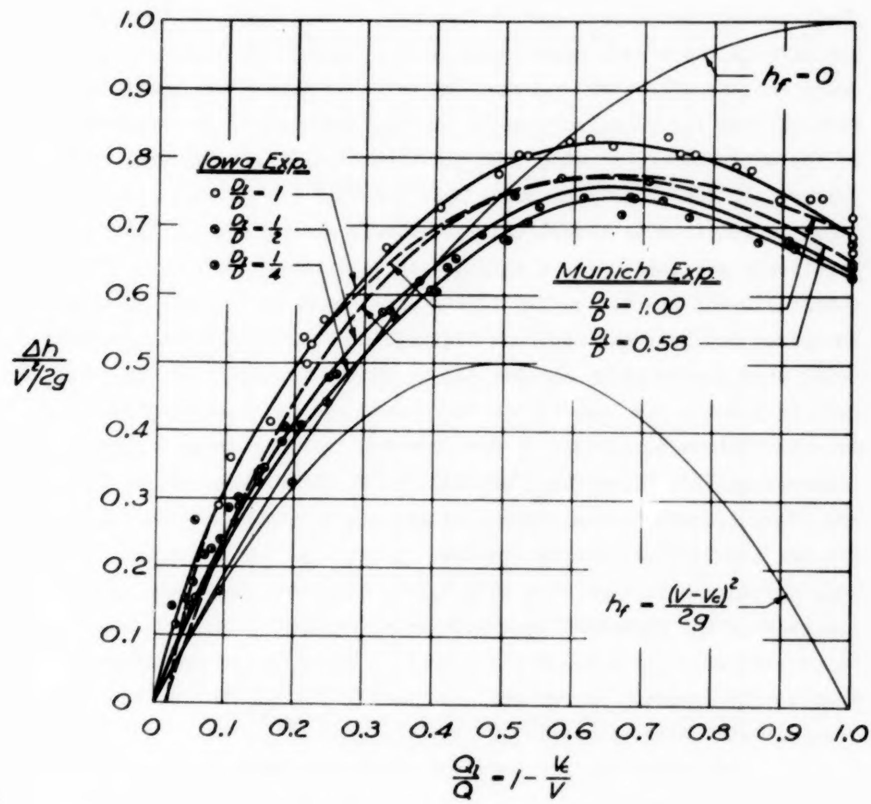

Fig. 4. Change in piezometric head in the conduit for dividing flow.

junction is in some ways comparable to that which occurs at an expansion. Evident from Fig. 4 are the facts that: (1) the gain in piezometric head is significantly large, (2) the head loss is never

as much as one-half of that at an abrupt expansion, and (3) for small values of the ratio Q_l/Q the head loss as computed is actually a sizeable negative quantity.

The systematic occurrence of negative losses was also observed by Vogel [5], Soucek and Zelnick [11], and Oakey [16], and cannot be ascribed to experimental error. A logical hypothesis is that the negative losses result from use of the square of the mean velocity in the energy equation without a correction factor to account for the effect of variation of velocity across the sections. Particularly for relatively small lateral discharges, the velocity of the water before flowing into the lateral is below average, both because the low velocities occur near the wall and because less force is required to divert the more slowly moving water. This hypothesis was tested in an approximate calculation, and resulting changes in the kinetic energy factor were found to be large enough to offset entirely the apparent negative losses. Confirming experiments were performed by Escobar [3] in which the region in the conduit from which the lateral discharge came was delineated by means of observations of streams of dye injected into the water a short distance upstream from the junction. As expected, the lateral flow was found to come from a segment of the conduit in which the velocity was significantly below average. Additional verification of this hypothesis is available in the fact that the larger the lateral the greater the segmental area from which the lateral could draw water with velocities below the average. Thus, as is indicated in Fig. 4, the apparent energy gain is greatest for $D_l/D = 1$ and decreases with decreasing size of lateral.

The head loss for the flow which goes from the conduit to the lateral is shown in Fig. 5, the velocity head in the lateral based on $V_l = Q_l/A$ being used as a reference for the upper part of the figure, and the conduit velocity V being used in the lower. Both are presented because the first is more useful for large discharge ratios and the second more useful for small ratios. The comparable results obtained by Vogel are also shown, together with

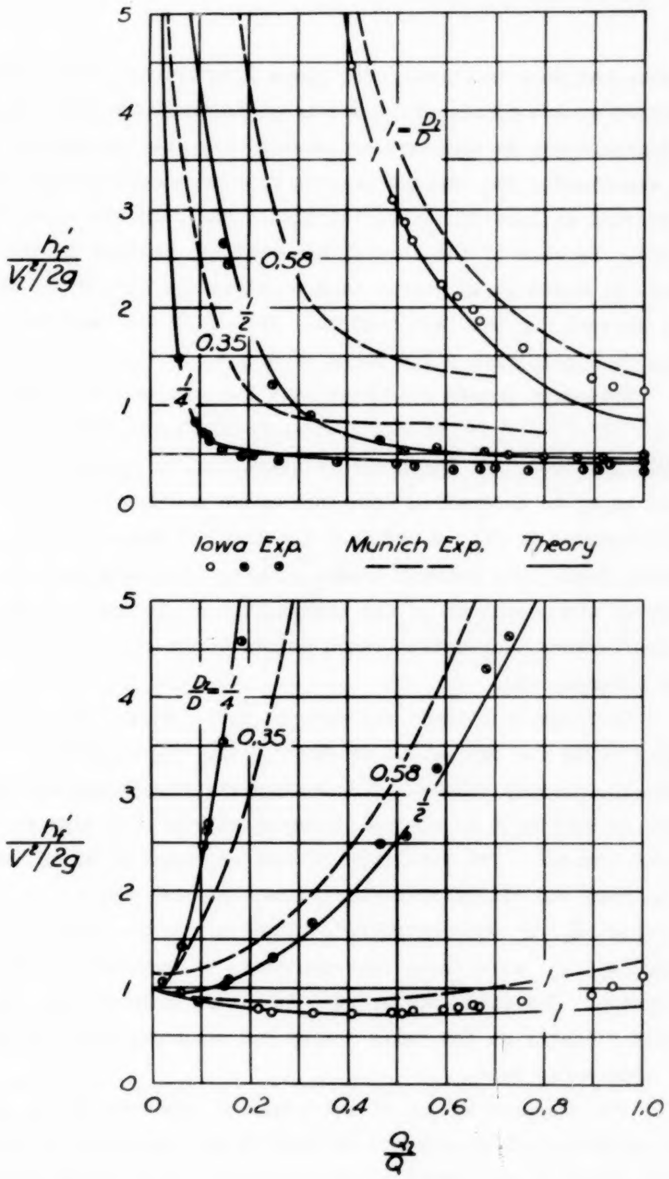

Fig. 5. Loss of head between the conduit and the lateral for dividing flow.

curves obtained from the theory for plane irrotational flow. The losses observed are considerably smaller than those recorded in the Munich experiments; no apparent explanation for this systematic difference was found. The remarkable correspondence between the results observed at Iowa and those calculated has been discussed elsewhere [14]. Because of the several assumptions implicit in the calculations, it would be an easier task to explain a difference than it is to account for the fact that only for $D_l/D = 1$ and for $Q_l/Q > 0.6$ do significant differences occur.

From experiments conducted with two successive 1-inch laterals, the effect of one lateral upon another a short distance downstream was assessed. If the pressure variation for a series of manifolds is to be determined from successive application of the results presented in Fig. 4 and from the known characteristics of the conduit resistance between branch points, some knowledge is necessary of the magnitude of the interaction or of the minimum separation between successive branch points which will cause a significant interaction.

At first an attempt was made to vary Q_l/Q and the corresponding ratio for the second lateral Q_l'/Q_c independently by adjusting the various valves. This method was so tedious and the variables so difficult to control independently that a simpler procedure was adopted. The two laterals were arranged so as to have identical loss and efflux characteristics, and only the overall discharge ratio Q_c'/Q was controlled. In his way the values of Q_l/Q and Q_l'/Q_c were those representative of successive laterals in any system. The value of Q_l'/Q_c is always greater than Q_l/Q, being only slightly so for small ratios but becoming twice as great as Q_c' approaches zero.

The measured values of the combined pressure change across the two branch points were compared with those computed from Fig. 4 for the various lateral spacings (4, 8, and 14 conduit diameters). The results are not well defined because the quantity sought was a second order difference, so that otherwise minor discrepancies caused

sizeable errors. From averages of the observed results, the measured recovery of piezometric head was found to be less than that obtained from Fig. 4, by about 15% for the 4-diameter spacing, by 10% for the 8-diameter spacing, and by less than 5% for the 14-diameter spacing.

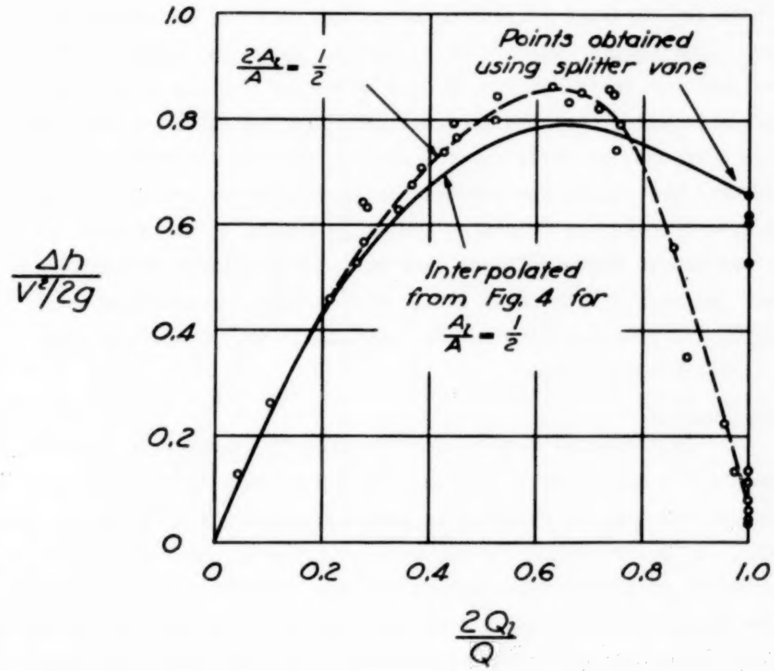

Fig. 6. Comparison of double and single laterals.

However, the values varied considerably with the discharge ratio. One can conclude that no significant interaction occurs if the spacing is 20 conduit diameters or more.

In a study of still another secondary effect, the character-

istics of a double lateral were determined. For the symmetrically placed 1-inch laterals, a comparison is made in Fig. 6 between the measured rise in piezometric head and a curve interpolated from Fig. 4 for $A_l/A = 1/2$. For values of $2Q_l/Q$ less than 0.25, the curve and data coincide. For values between 0.25 and 0.75, a larger recovery is found for the double lateral, presumably because of the increased opportunity for the water moving at low velocities to flow into one or the other of the laterals. Above 0.75 the observed values decrease rapidly with increasing discharge ratio in an unexpected manner. During the experiments for which Q_l/Q was large it was noted that the pressures and discharges varied rapidly over a large range, indicating that the flow was markedly unstable. It was concluded that unequal division of flow into the two laterals took place, and that rapid and periodic changes in this division caused excessive losses. In obtaining the solid points in the figure the pipe was capped and a splitter vane was placed between the entrances to the laterals. With equal and stable division of the flow thus assured, the pressure rise became essentially that predicted from the single-lateral tests.

Combining Flow

Experimental results for the drop in piezometric head in a conduit for combining flow were obtained by Niaz [2]. These are compared with Vogel's results, as revised by Kinne for $D_l/D = 1$, in Fig. 7. Quite similar trends were obtained in the two sets of experiments, although some differences are evident. Also included in the figure are two computed curves, one obtained from the Bernoulli and the other from the momentum equation. For the first the head loss was set equal to zero, and for the second the unbalanced force was set equal to zero.

The pressure drop is seen to vary from 1.5 to 2.5 times the change in kinetic energy. The unbalanced force, set equal to zero by Favre [13] for this type of flow, is seen to vary considerably with the diameter ratio. The force is sizeable in the downstream direction for $D_l/D = 1$ and about equal in magnitude but

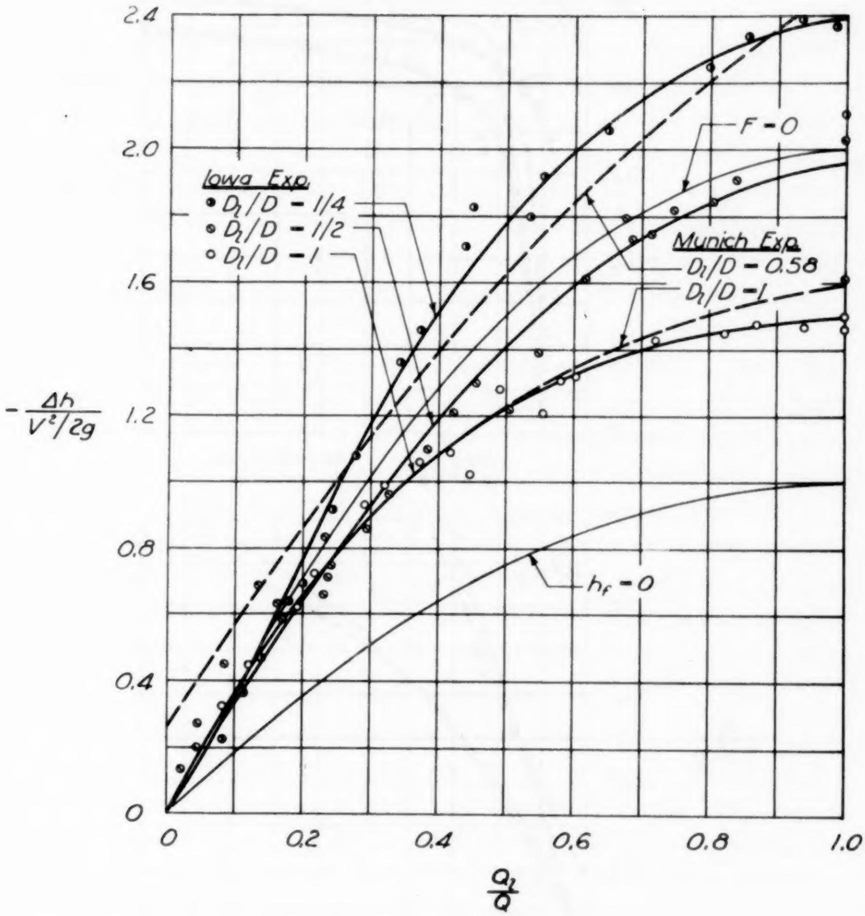

Fig. 7. Change in piezometric head in the conduit for combining flow.

in the upstream direction for $D_l/D = 1/4$. The marked difference arises from the fact that the water coming from the large lateral must be accelerated, whereas for the small lateral a very considerable

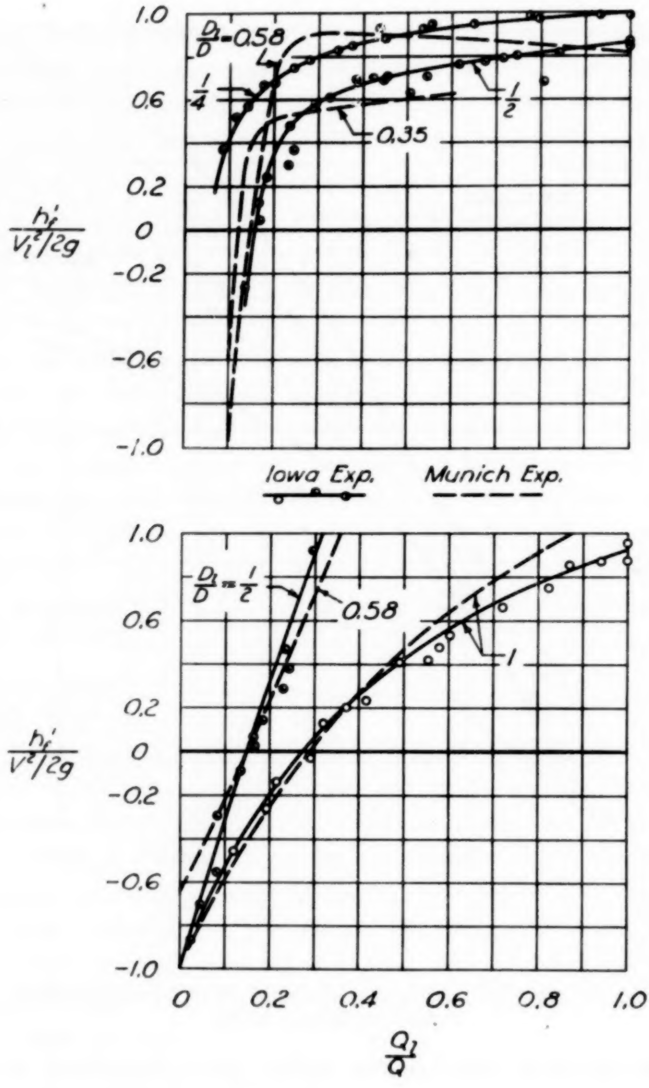

Fig. 8. Loss of head between the lateral and the conduit for combining flow.

deceleration takes place if Q_l/Q is nearly unity. In the comparison that Favre made with the Munich experiments, the apparent discrepancies were evidently assumed to be relatively unimportant.

For the loss of head experienced by the lateral flow, it was again thought advantageous to use two plots. As shown in Fig. 8, the head loss is most conveniently represented in its ratio to the velocity head in the lateral if Q_l/Q is large. For small values, and particularly for $D_l/D = 1$, the velocity head downstream from the junction is a more significant reference. Once again negative losses are seen to occur. These can be explained logically for the limiting case of $Q_l/Q = 0$, as pointed out by Vogel, since the pressure in the lateral is equal to that in the conduit at the junction but is at rest. Thus for very small lateral discharges a gain in energy takes place. Once again this gain is only apparent, as the water flowing from the lateral must go to a region where the velocity is below average. In any event, each of the curves approaches a value corresponding to $h_f' = -V^2/2g$ as $Q_l/Q \to 0$. In the upper plot this would correspond to very large negative ratios as V_l is necessarily much less than V if Q_l/Q is sufficiently small.

DISCUSSION OF RESULTS

From the foregoing comparisons of observed and theoretical trends, it is evident that the characteristics of flow through manifolds must be predicted empirically, in part at least. Only for the one case of loss for the lateral efflux in dividing flow was good correspondence found between results predicted from a simplified theory and those observed in the laboratory. Even in this case the concurrence appears to be somewhat fortuitous.

In the several plots the variations of the piezometric and total head are indicated for the circular smooth conduits tested both at Iowa and at Munich. As these are for only one of a wide variety of possible geometrical arrangements, the results presented herein may well differ from those which would be obtained for other forms. Several modifications were tested at Munich, including modified lateral inlet sections and laterals making angles of 45°

and 60° with the conduit. Also, in studies of typical lock and gas-burner manifolds, the characteristics of other geometrical forms have been determined. In most cases, the general trends noted in this study are the same as those for other manifold arrangements.

For estimation or for some designs, the results presented herein may prove to be satisfactory. In other projects, in which more accurate results are essential, a typical junction can be studied in the laboratory and the results obtained can then be used for the design of an entire manifold system. Soucek and Zelnick followed this procedure and showed its effectiveness in studies of Panama Canal locks. A difficulty in the direction application of the results for a single branch point is that trial calculations beginning with the outlet farthest downstream must be made and a final check is obtained only if the computed head at the upstream end corresponds with the design value. To avoid such trial solutions, Carstens [17] and Yanes [4] presented graphical methods which can be used directly. Carstens used the data presented in Fig. 1 in deriving a general design procedure for the distribution networks of sprinkler irrigation systems. Yanes devised a similarly general method of computing the flow for lock manifolds once the characteristics of a single port have been determined.

CONCLUSION

In a determination of the principal characteristics of the flow of fluids at a branch point in a conduit, the various effects are most logically treated separately. Changes in piezometric and total head have been presented for both combining and dividing flow, and for each case the two component parts of the flow have been submitted separately. The various occurrences have been interpreted with reference to conventional energy and momentum equations, indications being made of the magnitude of the head-loss term and of the resultant pressure on the wall of the lateral. The effect of lateral spacing was evaluated, and the characteristics of a symmetrical double lateral were found to coincide satisfactorily with those for a single lateral having the same effective area ratio.

should be small compared to the equivalent shunt resistances of the transformers, and large compared to the winding resistances, the optimum compromise probably being reached with resistances of the order of 50 to 300 ohms. The computing transformers are connected to yield the required turns ratios, after which they are wired to the selected rheostats, which have been set to the proper values. The "grounds" or datum points are connected together.

3. Load currents are supplied by the voltage sources in series with resistances, the voltage drop across the resistances serving as an indication of current. The scale factor between force and moment and the corresponding currents is a matter of convenience.

4. The branch currents and joint voltages are measured. The former represent force or moment, which are readily converted to stress, and the latter represent slope or deflection. If the structure is seen to be of unsatisfactory design, the cross-section of selected members can be altered by simply changing the setting of the associated rheostats, this usually necessitating a readjustment of the load currents. Measurements are then made as before, the whole process being repeated until the design is satisfactory.

5. A rapid numerical check can be made by noting whether the currents leaving each joint and the voltage drops around each mesh sum to zero. This, however, does not reveal errors in the circuit diagram. A more thorough check can be conducted in purely structural terms, by noting whether the results satisfy the requirements of equilibrium, and whether the deflections and slopes are mutually compatible.

Experimental Results. The structures of Figs.2, 8, 9 and 10 were solved experimentally. The error is expressed as a percentage of the maximum values encountered, which is considered more indicative of the overall accuracy than the error expressed as a percentage of each individual quantity. The latter can be much larger than the former, for members which are relatively lightly loaded. For these members, however, the error is of far less importance than for the heavily loaded members.

The computing transformers were uncompensated, except that the winding resistances were subtracted from analogue resistances where possible.

The units used were as follows: resistance in ohms, voltage in divisions of the ten-turn rheostat, and current in divisions per ohm. In the present experiments, 1000 divisions represented six volts.

Fig.2(c). The branch resistance R was 100 (200 for branch 2-5), and F was 6.58. The joint voltages (from which the branch currents are readily calculated) were measured, the greatest error compared to theoretical values being 2% of FR.

Fig.8. The tensile compliance was represented by 100 ohms. For L equal to unity, the bending compliance resistor ρ would have been 1200 ohms, which was considered too high compared to the transformer equivalent shunt resistance of 7500 ohms. Hence L was set at two, thus reducing ρ to 300, transformer T_1 then having a 2:1 turns ratio. The value of F was 7.94. The various currents were measured, the greatest error being 1.5% of F.

Fig.9. The bending compliance resistor was 300 ohms for member 1-2.

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